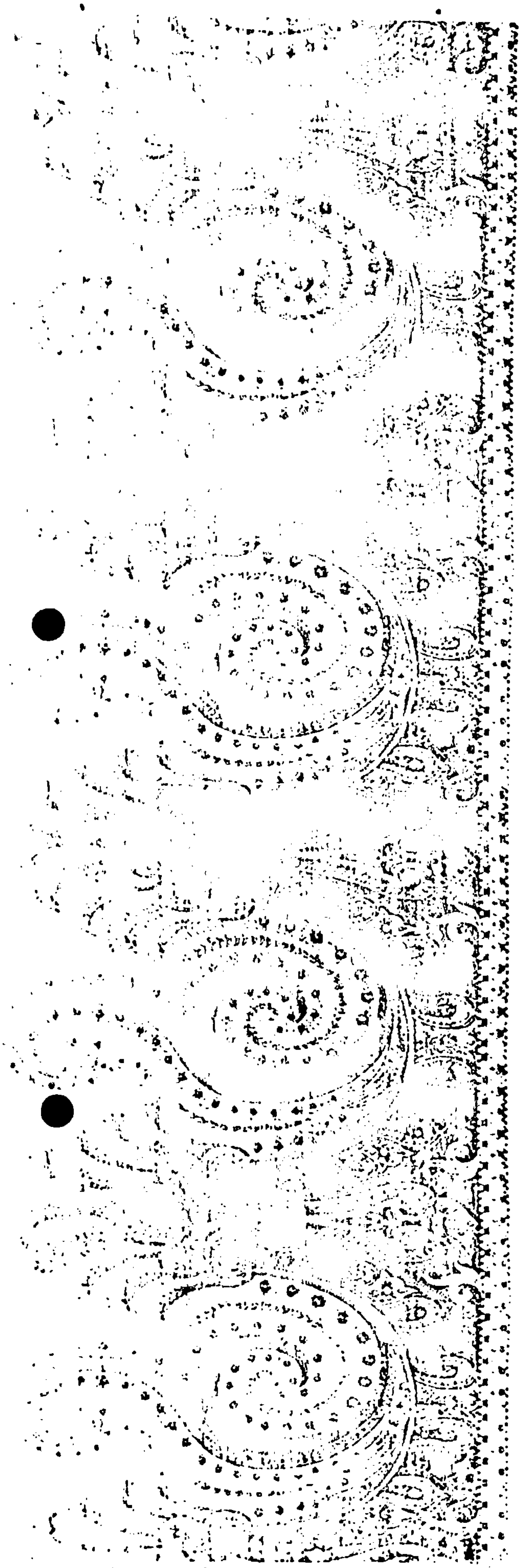
A Traditional Paisley Shawl Pattern

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CRITICAL LOADING OF SURFACE DISCONTINUITY IN LAYER INTERFACE

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INTRODUCTION

Composite systems often fail due to the loss of bond between individual layers or phases. As a result of technological processes the bond between the individual layers in a composite is considerably variable and it is not an infrequent phenomenon that this bond tends, in a certain limited area, towards zero. A number of defectoscopic methods makes it possible to identify such material discontinuities and, indeed, spots with an existing internal two dimensional crack even before the system had begun to be used /loaded by external loads/ were described in literature for concrete systems +/. In a certain state of stress such two-dimensional discontinuities may and indeed do, develop so much as to cause local loss of stability of one component or a flexural failure of a certain layer or even a complete separation of both originally connected components.

There is a great number of causes of origin of two-dimensional discontinuities /defects/. In the case of glass-reinforced plastics, for example, their number includes insufficient pressure or temperature of pressing or, on the contrary, temperature exceeding the critical temperature of decomposition of one of the components, presence of an undesirable fluid phase /either liquid or gaseous/ beneath the impervious coating of a pervious material, such as concrete, concentrating, for example, in the proximity of pores only. The present defects of the interface may have rounded ends /as is the case of pores/ or sharp ends which result in high concentration of stresses, when the stress field changes, initiating the propagation of the defect and a further damage.

A theoretical assessment of the state of stress or strain in the vicinity of such discontinuities, even though only approximate, may in many cases provide an adequate idea about the propagation of defects in the case of a change of external conditions and, consequently, on the life expectancy of the system. Such assessment becomes particularly important, if the impervious coating layer is subjected to an internal overpressure, such as, the pressure of a gas or a liquid, passing through the lower, pervious layer. A typical example of such a system is a concrete structure, protected against the effect of environment by a protective coating, insulating layer or cladding.

We shall show how it is possible to assess the circumstances of propagation of a two-dimensional crack of this type.

+/ between the individual layers of GRP, between the outer faces and the core of sandwich systems, between the protective coat or core and the load-bearing part, etc..

PRINCIPLE OF SOLUTION

For the assessment of the stability of defects the theory of balanced cracks, foundations of which were laid by Griffith[1] and which was further developed by Irwin[2] and Barenblatt[3] is mostly used. The assessment of the balanced state of cracks at the interface of two materials was considered particularly by Erdogan and Arin[4], Chow and et al.[5]

The theoretical model of a crack /Fig.1/ assumes that apart from its ends the juxtaposed crack faces have no mutual links, and that the length of their tips d, where cohesion /valence/, but also adhesion /intermolecular/ bonds act, is small in comparison with its total length 2a. This distribution follows out of the knowledge that the shape of the faces at the head of the crack does not depend on external load, but is determined by the properties of the material with regard to the effect of cohesion forces in this locality. It is further assumed that the change d in the course of loading equals zero and that at the moment of failure the magnitude of cohesion forces at the head of the crack attains the value of theoretical strength of the material.

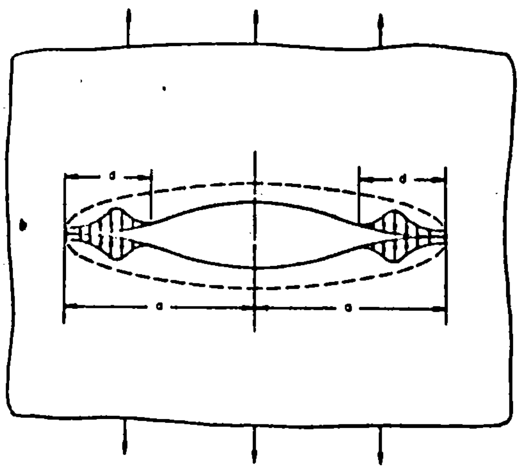

Fig.1.

These assumptions, naturally, determine and limit the suitability of problems which can be solved on the basis of balanced crack theory. The coincidence of assumptions with reality deserves particular attention in the cases whose solution permits further simplification. If, for example, the linear dimension of the defects is higher approximately by one order than the thickness of the covering layer, it can be assumed that almost all strain energy is used up by the flexure or expansion of the layer above the crack. In such a case it is possible to determine the critical load pertaining to the given crack /or the critical crack length pertaining to the given load/ by the method based on Griffith theory, applied by Gilman[6], who determined the effective surface energy γ_I on the basis of data obtained from the splitting of a sample in the form of a double cantilever beam.

If we consider a two-dimensional circular defect at the interface of a thin elastic plate with a rigid base, it is possible to calculate the strain energy of a thin /thickness = t/ circular plate /i.e. if its diameter $2r \geq 6t$/ on the basis of Kirchhoff theory of plates and, as will be proved further on, to determine the effective surface energy on the basis of an equivalent one-dimensional beam with an limited crack at midspan.

STATES OF EQUILIBRIUM OF A LINEAR AND A CIRCULAR CRACK

The application of Gilman's method of determination of effective surface energy γ_I is shown on an example of an equivalent beam /Fig.2/. To comply with the assumption of the rigid base within the limits of accuracy of a technical calculation, it is necessary that the ratio of the heights h_p and h_d complies with the relation[7]

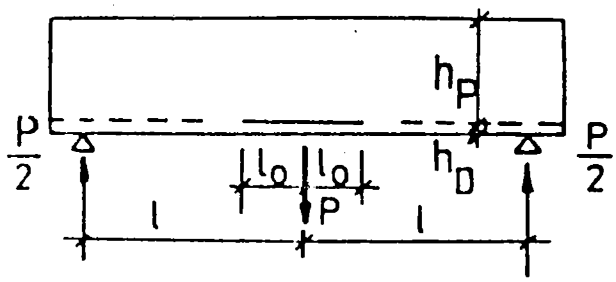

Fig.2.

$$\frac{h_p}{h_d} > 10^3 \sqrt{\frac{E_d}{E_p}} \qquad /1/$$

where E_d and E_p are moduli of elasticity of the materials of the thin plate strip and the rigid base respectively.

The basic relation of the Griffith energy method is the energy balance of all processes participating in the crack propagation in accordance with the equation

$$E = S + U - W \qquad /2/$$

where E is the overall energy of the body, S the energy of the surface of the formed crack, U the strain energy of the plate strip and W the work of external forces.

Since it is possible to assume for the elastic structures that $W = 2\,U$, Eq. /2/ will obtain the form of

$$E = S - U \,. \qquad /3/$$

The state of equilibrium of a cracked body is now determined from the extreme of the overall energy E, determined by the condition

$$\delta E = 0 \,. \qquad /4/$$

Let us follow now two particular cases, viz. a linear and a two-dimensional crack.

a/ The strain energy of a thin plate strip considered as constrained at the ends of an limited crack /according to Fig.2/ is

$$U = \frac{P^2\, a^3}{48\, E_d\, I_d} \qquad /5/$$

where I_d is the moment of inertia of the beam.

The surface energy of the crack is determined by the expression

$$S = 2\, \gamma_I\, ba, \qquad \gamma_I = \gamma_I' + \gamma_I'', \qquad /6/$$

where γ_I is the specific mean surface energy of upper / γ_I'/ and lower / γ_I''/ crack faces for the crack opening of the 1st mode +/, and $\underline{b}$ is the beam width.

After the substitution of Eq. /5/ and Eq. /6/ in Eq. /3/ we obtain the expression for the overall energy of the body, viz.

$$E = 2\ \gamma_I\ ba - \frac{P^2\ a^3}{48\ E_d\ I_d} \qquad /7/$$

and from its variation according to $\underline{a}$, using Eq. /4/ we obtain the condition

$$2\ \gamma_I\ b - \frac{P^2\ a^2}{16\ E_d\ I_d} = 0 \qquad /8/$$

from which it is possible to determine the critical value P_{crit}:

$$P_{crit} = \sqrt{\frac{32\ \gamma_I\ b\ E_d\ I_d}{a}} \qquad /9/$$

or the critical length a_{crit}:

$$a_{crit} = \sqrt{\frac{32\ \gamma_I\ b\ E_d\ I_d}{P^2}} \qquad /10/$$

or the specific surface energy

$$\gamma_I = \frac{P^2\ a^2}{32\ b\ E_d\ I_d} = \frac{M_o^2}{2\ b\ E_d\ I_d} \qquad /11/$$

where M_o is the moment in constraint.

b/ Analogously it is possible to determine the critical values of the load, or the critical dimension of a circular crack /Fig.3/. For the frequent practical case of loading by internal pressure of vapours or liquid or the effects of temperature it is possible to base the calculations on the deformation of a circular plate perfectly constrained along its circumference, loaded by a full uniformly distributed load $\underline{p}$.

The strain energy is determined by the expression /8/

$$U = \frac{p^2\ R^6}{6\ .\ 64\ D} \qquad /12/$$

where D is the flexural rigidity of the plate

$$D = E_d\ h_d^3\ /\ 12\ /1 - \nu^2/.$$

+/ 1^{st} mode - crack faces displacements are normal to the initial crack surfaces.

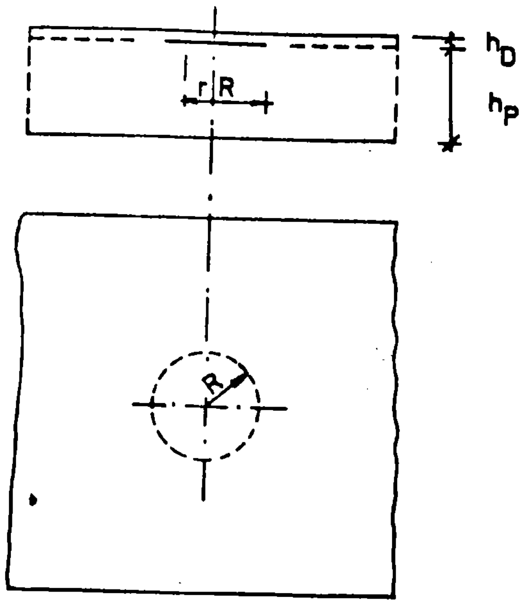

Fig.3.

The surface energy of the circular crack is determined analogously with Eq. /6/ as the product of the circle area and the specific surface energy γ_I, viz.

$$S = \gamma_I \; \pi R^2 \; . \qquad /13/$$

From the extreme of Eq. /4/ according to R we obtain, after rearrangement, the condition for critical values of p_{crit} and R_{crit}

$$\gamma_I - \frac{p^2 \; R^4}{128 \; D} = 0 \qquad /14/$$

from which we obtain

$$p_{crit} = \frac{16}{R^2} \sqrt{\frac{\gamma_I \; D}{2}} \qquad /15/$$

or

$$R_{crit} = 4 \sqrt[4]{\frac{\gamma_I \; D}{2 \; p^2}} \qquad /16/$$

or the specific surface energy of the crack γ_I

$$\gamma_I = \frac{p^2 \; R^4}{128 \; D} \qquad /17/$$

If we use, as a test for the determination of γ_I, the flexure of the beam, it is possible, by a comparison of Eq. /11/ and Eq. /17/, to express the critical values of the circular crack by means of the critical values of the linear crack. The comparison of both equations yields

$$\frac{P^2_{crit} \; a^2}{32 \; b \; E_d \; I_d} = \frac{p^2_{crit} \; R^4}{128 \; D} \qquad /18/$$

If we consider further R = a and substitute $D = \dfrac{E_d \; I_d}{/1- \nu^2/b}$, we obtain after re-arrangement,

$$p_{crit} = \frac{2 \; P_{crit}}{ab \; \sqrt{/1- \nu^2/}} \qquad /19/$$

EXPERIMENTAL RESULTS AND PRACTIVAL APPLICATIONS

To verify the correctness of the proposed method of calculation of the critical load /or the critical dimension/ of the defect two series of test samples were made, comprising three samples each.

The first series of test samples were beams of 20 cm x 32 cm x 90 cm size, consisting of a precast resin concrete slab 2 cm thick, provided with fine gravel spread at the contact face, and an additionally concreted concrete part 30 cm thick. At midspan a separating polyethylene sheet, size 10 cm x 20 cm was placed at the interface to create an artificial defect.

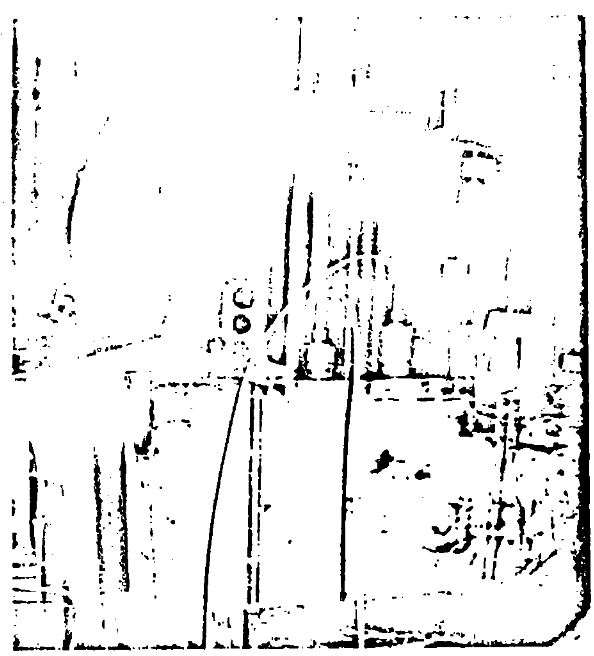

Fig.4.

The arrangement and the process of the test to separate both materials are shown in Fig. 4. For the above mentioned defect the following critical strength values were obtained:

Sample No.	1	2	3
P_{crit} /N/	1800	2000	2100

The other series of test samples consisted of concrete blocks size 90x90x30 cm provided with a precast resin concrete slab 2 cm thick. Fig. 5 shows the resin concrete slab with a 20 cm dia. separation sheet and a tube for the supply of pres surised oil before concreting. The inner overpressure in the artificial defect was produced by a two-stage system air-oil which guaranteed the uniform distribution of pressure in time /Fig.6/. For the given diameter of the defect the following critical pressures were ascertained:

Sample No:	1	2	3
p_{crit} /MPa/	0,60	0,45	0,50

After the substitution of the critical force value in Eq. /19/ we obtain, for the test samples of the second series, i.e. for the blocks with a dia. 20 cm circular defect, the critical pressure of p_{crit} = 0,42 MPa which is in a good conformity with the measured critical pressure value.

This has proved that the effective surface energy of a two-dimensional defect can be determined, with a very good accuracy, on the basis of a simple test of a beam with a linear defect.

The crack development was observed by the outflow of pressurized oil through bored openings situated on a 26 cm dia. circle concentrical with the artifical defect /see Fig.7/. Fig.8 shows the face of the artificial defect after a removal of the whole precast resin concrete slab.

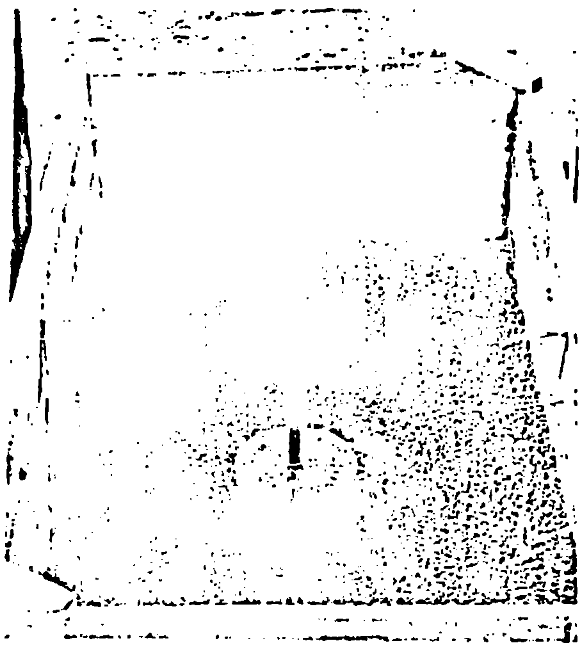

Fig.5.

Fig.6.

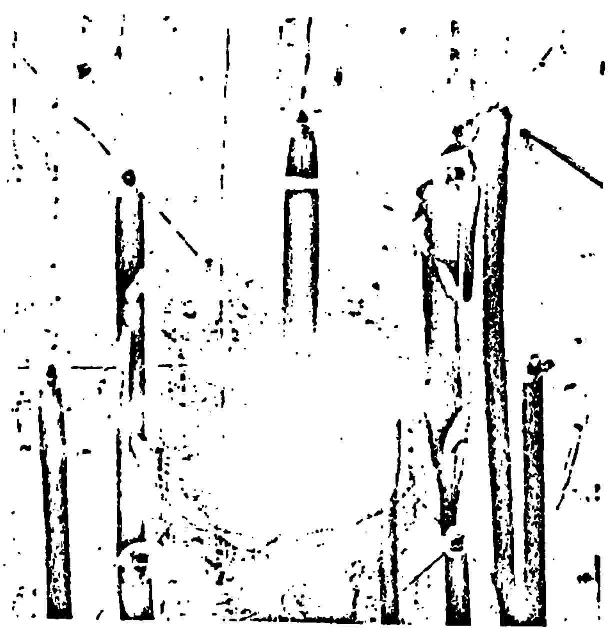

Fig. 7.

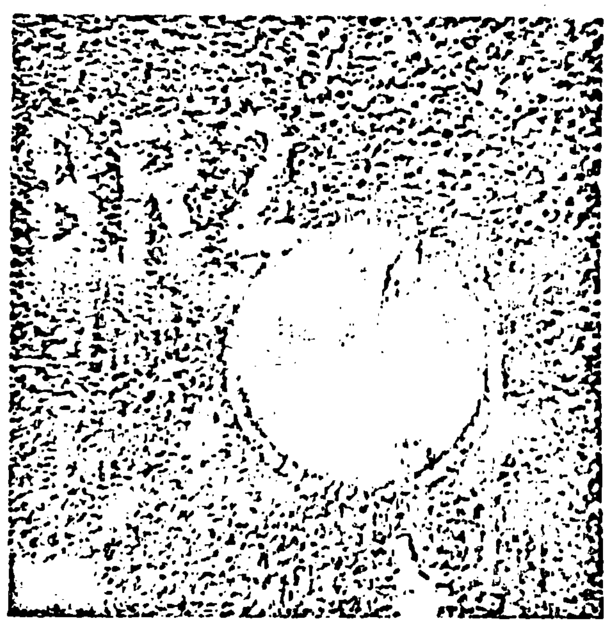

Fig. 8.

CONCLUSION

The study has proved that a relatively simple method can be used for the forecasting of a critical load of a two-dimensional crack at the interface of two materials, which is important particularly in the cases of loading of a surface /impervious/ layer by inner overpressure, or for the determination, for the given load /overpressure/, of the critical /maximum permissible/ magnitude of the defect at the interface resulting from technological or other production defects.

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